# Concatenated Differential Space-Time Block Codes and Four Dimensional 8-PSK Trellis Coded Modulation for Wireless MIMO system with Rayleigh fading Noise

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*Abstract*— In this paper, new concatenated inner-differential space-time block code (DSTBC) and outer four-dimensional 8-PSK trellis coded modulation (4D-8-PSK-TCM) encoder is designed for wireless MIMO system with Rayleigh fading noise in the absence of channel state information (CSI). The proposed concatenated encoder takes advantage of both the high coding gain from its outer-coder and the ease of the detection process from its inner-coder. Simulation results demonstrate the good performance of the proposed scheme (4D-8-PSK-TCM-DSTBC) against the well-known DSTBC scheme for the same spatial diversity and similar signal constellation. The 3 dB performance gap usually involved by symbol by symbol differential detection is also observed between the presented non-coherent detection method and the coherent detection (4D-8-PSK-TCM-STBC) recently published.

## *Keywords*— Multidimensional TCM, fading channel, differential space-time block code.

### I. INTRODUCTION

Since invented by Alamouti [1], space-time block code (STBC) has sparked wide interest as it promises to significantly increase transmission rates in mobile radio communications [2]-[3]. The key characteristic of STBC is exploiting both time and antenna diversity which is an effective technique for mitigating the impairments of wireless fading channels. Moreover, STBC improves the system capacity and provides diversity gain. When no knowledge of CSI is available about the flat fading channel parameters, the STBC scheme requires specific estimation methods. These estimation techniques should enable the receiver to estimate the channel characteristics accurately. Then, the differential space-time block code firstly introduced by Tarokh [4] is used here since this coding technique does not require any channel estimation at the receiver side but is done through mutual compensation of channel errors between successive received symbols only.

Unfortunately, the differential STBC does not achieve any additional coding gain [4]. Therefore, the DSTBC scheme is used here in conjunction with a channel coder that confers to the proposed scheme a large coding gain over fading channels. Recently, Low-density Parity-check (LDPC) and the conventional TCM scheme were introduced as an outer coder in conjunction with DSTBC code in order to achieve a coding gain [6]-[8]. According to [9], the conventional trellis coded modulation operating with a multiple-input multiple output (MIMO) system represents a powerful outer code.

In our present work, we attempt to extend the concatenated traditional TCM with the well-known DSTBC by considering multidimensional trellis coded modulation (multi-D TCM) as outer-coder which is known to provide high bandwidth efficiency. According to [10], multi-D TCM stands an attractive candidate for outer-codes in a concatenated STBC system. In fact, we propose here a reliable communication system that concatenates DSTBC with four-dimensional trellis coded phase shift keying modulation scheme over quasi static Rayleigh channel. A symbol interleaver is also introduced in the transmission chain in order to reduce the effect of burst error due to fading. We focus here on the wellknown 4D-8-PSK-TCM scheme described in [11]. In this work, the channel characteristics are assumed to be unknown by the receiver. Simulation results prove the efficiency of concatenating the interleaved streams of this trellis code with DSTBC and clearly show the BER performance compared to the non-coded system with a reasonable growth of complexity.

The remainder of this paper is organized as follows. After presenting the new communication scheme using the wellknown DSTBC concatenated to 4D-8-PSK-TCM with no CSI at the receiver side, in Section 2, the 4D-8-PSK-TCM encoder is described in Section 3. In Section 4, the conventional DSTBC is discussed. Then the performance analysis of the novel method is given in section 5. Finally, Section 6 is devoted for the conclusions drawn from this work.

#### II. SYSTEM MODEL

As shown in Fig. 1, at the transmitter side, the information bits are encoded through the four dimensional trellis coded modulation. Then the output symbols of the 4D-8-PSK-TCM encoder are interleaved by a matrix symbol interleaver. The interleaved symbols are then coded by DSTBC block equipped with two transmit antennas. We assume that there are Nr receive antennas, so that the diversity order is defined as  $K = 2 \times N r$ .

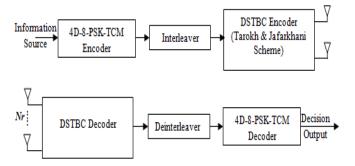


Fig. 1 Block diagram of a MIMO system concatenating DSTBC and 4D-8-PSK-TCM, Nt = 2; Nr

Assuming that the fading channel coefficients are unmovable during 2T, where T is the symbol duration, then the received signals over two consecutive symbol periods can be expressed by:

$$R = H \cdot S + N \tag{1}$$

Where S is the transmitted symbols vector. The vector N is the temporally i.i.d. zero-mean complex Gaussian noise vector. H denotes the channel transfer matrix. For a MIMO scheme with two transmit antennas and Nr receive antennas, the channel matrix H can be written as:

$$H_{Nr\times2} = \left[h_{ij}\right], (i=1...Nr, j=1,2)$$
(2)

Each element of the matrix H, denoted by  $h_{ij}$ , represents the complex path from transmit antenna j to the receive antenna i. The channel noise model is flat Rayleigh fading with no spatial correlation between transmit antennas. The MIMO fading channel is also assumed to be static during two consecutive transmission slots. These fading coefficients are modeled as Rayleigh process whose variance is equal to 0.5 with uniformly distributed phase in the interval.

#### III. DESIGN OF MULTIDIMENSIONAL TRELLIS CODED MODULATION FOR FADING CHANNEL

Differential STBC does not provide any coding gains. In order to achieve better performance over fading channels, concatenated 4D-8-PSK-TCM coder with DSTBC represents potential viable option. In fact, multidimensional trellis-coded modulation is known to achieve good coding gain without anv bandwidth expansion. Note that optimal multidimensional TCM concatenated to STBC scheme are optimal in perfect CSI knowledge cases also stand optimal when concatenated to non-data-aided DSTBC scheme. Divsalar and Simon [12]-[13] show that the design criteria of multidimensional M-PSK TCM codes for fading channels are the effective code length over span two symbol intervals and the product-sum distance over a span of two symbol intervals. Since we consider a concatenation of multidimensional M-PSK TCM scheme in conjunction with DSTBC, optimal trellis code contrived for fading channels is considered. The outer coder as shown in Fig. 2 is 4D-8-PSK-TCM with 2 bits/symbol spectral efficiency designed by the CCSDS. The considered trellis outer code has a rate of <sup>3</sup>/<sub>4</sub> and a constraint length v = 7 and consequently has 64 states.

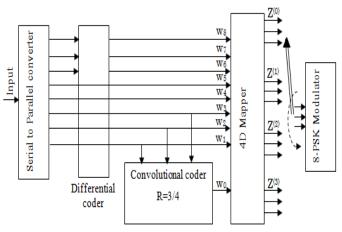


Fig. 2 4D-8-PSK-TCM coder for 2 bits/symbol spectral efficiency.

The 4D mapper implements a logical mapping described by the following equation.  $\begin{bmatrix} & 0 \\ 0 \end{bmatrix}$ 

$$\begin{bmatrix} Z^{(0)} \\ Z^{(1)} \\ Z^{(2)} \\ Z^{(3)} \end{bmatrix} = \left[ \left( 4w^{(8)} + 2w^{(7)} + w^{(6)} \right) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \left( 4w^{(5)} + 2w^{(4)} + w^{(3)} \right) \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + \left( 4w^{(2)} + 2w^{(1)} + w^{(0)} \right) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right] mod(8)$$

$$\left( 4w^{(2)} + 2w^{(1)} + w^{(0)} \right) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} mod(8)$$

The correspondence between the signals at the input of the modulator and the 8-PSK phase states of the constellations is set by reference to the conventional mapping. This equation shows that only the bits which are sensitive to phase rotation ( $\pi/4$  here) are differentially encoded.

#### IV. DIFFERENTIAL SPACE-TIME BLOCK CODES

In this section, we will briefly introduce the encoder and decoder of differential STBC [4], firstly introduced by Tarokh and Jafarkhani, which is based on the Alamouti transmit diversity scheme.

#### A. Encoding Scheme

For two transmit antennas, this scheme starts by sending an arbitrary initial signal vector  $S_0 = (s_1, s_2)$  unknown to the receiver. According to the Alamouti STBC, the transmitter sends signals  $s_1$  and  $s_2$  at time one simultaneously, and signals  $-s_2^*$  and  $s_1^*$  at time two. This initial vector does not carry any information data. Then the transmitter encodes the data sequence in a differential manner. Assume in the  $(t-1)^{th}$  encoding operation, the signal vector fed to the STBC encoder is  $S_{t-1} = (s_{2t-1}, s_{2t})$ . Therefore, the symbols  $s_{2t-1}$  and  $s_{2t}$  are sent from transmit antennas 1 and 2, respectively, at time 2t -1, and that signals  $-s_{2t}^*$  and  $s_{2t-1}^*$  are sent from transmit antennas 1 and 2, respectively, at time 2t. Then, the procedure of the DSTBC coding forms an orthonormal basis denoted by:

$$D(S_{t-1}) = \begin{bmatrix} s_{2t-1} & s_{2t} \\ -s_{2t}^* & s_{2t-1}^* \end{bmatrix}$$
(4)

Where  $D(S_{t-1})$  is the reference to calculate the next STBC encoder input  $S_t = (s_{2t+1}, s_{2t+2})$  as:

$$S_{t} = R_{t} D\left(S_{t-1}\right) \tag{5}$$

Where  $R_t = (R_t^1, R_t^2)$  is the differential coefficient vector which is used to carry message.

At time 2t + 1, a block of 2*b* bits, denoted by  $B_{2t+1}$ , reaches the encoder. The first *b* bits are mapped into a constellation symbol  $a_3$  and the second *b* bits are mapped into a constellation symbol  $a_4$  using Gray mapping. The transmitter apply a mapping *M* to the block  $B_{2t+1}$  in order to compute the two complex coefficients  $R_t^1$  and  $R_t^2$ . This mapping *M* is defined by :

$$M\left(B_{2t+1}\right) = \left(R_{t}^{1}, R_{t}^{2}\right) = \begin{cases} R_{t}^{1} = a_{3}a_{1}^{*} + a_{4}a_{2}^{*} \\ R_{t}^{2} = -a_{3}a_{2} + a_{4}a_{1} \end{cases}$$
(6)

Where  $a_1 = a_2 = \frac{1}{\sqrt{2}}$ . Then, the encoder computes:

$$(s_{2t+1}, s_{2t+2}) = R_t^1(s_{2t-1}, s_{2t}) + R_t^2(-s_{2t}^*, s_{2t-1}^*)$$
(7)

The transmitter then sends  $s_{2t+1}$  and  $s_{2t+2}$ , respectively, from transmit antennas one and two at time 2t+1, and  $-s_{2t+2}^*$ ,  $s_{2t+1}^*$  from antennas one and two at time 2t+2. This process is inductively repeated until the end of the frame.

#### B. Differential decoding

For simplicity, we consider only one antenna at the receiver side. We denote  $r_i$  the received signal and  $n_i$  the noise sample at time t. The received signals at times 2t - 1, 2t, 2t + 1 and 2t + 2 can be represented by:

$$r_{2t-1} = h_{11}s_{2t-1} + h_{12}s_{2t} + n_{2t-1}$$

$$r_{2t} = -h_{11}s_{2t}^{*} + h_{12}s_{2t-1}^{*} + n_{2t}$$

$$r_{2t+1} = h_{11}s_{2t+1} + h_{12}s_{2t+2} + n_{2t+1}$$

$$r_{2t+2} = -h_{11}s_{2t+2}^{*} + h_{12}s_{2t+1}^{*} + n_{2t+2}$$
(8)

As in DSTBC, two combined decision signals  $\tilde{R}_{t}^{1}$  and  $\tilde{R}_{t}^{2}$  can be constructed as follows:

$$\widetilde{R}_{t}^{1} = [r_{2t+1} \quad r_{2t+2}^{*}][r_{2t-1} \quad r_{2t}^{*}]^{H} 
\widetilde{R}_{t}^{2} = [r_{2t+1} \quad r_{2t+2}^{*}][r_{2t} \quad -r_{2t-1}^{*}]^{H}$$
(9)

According to [4], the combined decision signal vector  $\left(\tilde{R}_{t}^{1}, \tilde{R}_{t}^{2}\right)$  can be expressed as:

$$\left(\tilde{R}_{t}^{1}, \tilde{R}_{t}^{2}\right) = \left(|h_{11}|^{2} + |h_{12}|^{2}\right)\left(R_{t}^{1}, R_{t}^{2}\right) + \left(\tilde{n}_{1}, \tilde{n}_{2}\right)$$
(10)

Where  $(\tilde{n}_1, \tilde{n}_2)$  is a noise vector.

As proved in [4], if no outer coder is added, the receiver computes the closest vector to  $(\tilde{R}_{i}^{1}, \tilde{R}_{i}^{2})$  then the reverse mapping M is applied. In this work, the DSTBC coder is concatenated to a 4D-8-PSK-TCM channel coder. Therefore, the signal vector  $(\tilde{R}_{i}^{1}, \tilde{R}_{i}^{2})$  is directly mapped by the reverse mapping M, denoted by M<sup>-1</sup>. The outputs of M<sup>-1</sup> are then decoded using the well-known Viterbi decoding algorithm.

The same procedure stands efficient for larger receive antenna number. For each receive antenna j, we compute  $\left(R_{t}^{1}\right)^{j}$  and  $\left(R_{t}^{2}\right)^{j}$ , using the same algorithm as for  $R_{t}^{1}$  and  $R_{t}^{2}$  given above. Then the input vector of the reverse mapping becomes  $\left(\sum_{j=1}^{Nr} \left(R_{t}^{1}\right)^{j}, \sum_{j=1}^{Nr} \left(R_{t}^{2}\right)^{j}\right)$ .

#### V. SIMULATION RESULTS

In this section, we provide simulation results for measuring the performance of the proposed scheme. The fading gain is assumed to be constant over each frame but varies from one frame to another. Assuming no CSI knowledge at the receiver side, concatenated channel coder and differential orthogonal space-time block codes (DOSTBC) is considered. For all the simulations, the outer code is assumed to be the four dimensional trellis coded modulation scheme, 4D-8-PSK-TCM, with 2 bits/symbol spectral efficiency.

In Figs. 3 and 4, simulation results for both the performance of the coherent STBC, the DSTBC, the coherent system 4D-8-PSK-TCM-STBC which is recently studied in [10] and the proposed system with no interleaving are depicted. Note that the spatial diversity of both systems is the same in each figure. In Fig.3, we evaluate the performance of the MIMO system with 2 transmit and 1 receive antennas on quasi static Rayleigh channels, but in Fig. 4, MIMO 2x2 system is considered. We assume that the 8-PSK modulator is also used in these cases. From these figures, we can see that the performance curves of the differential schemes are parallel to those of coherent schemes, which indicates that the differential schemes also achieve full transmit diversity due to the orthogonal designs. However, the receiver does not require any channel state information; the differential schemes are 3 dB less than STBC codes with coherent detection. This 3 dB penalty can also be physically justified by the doubling of noise power for the differential detection. From these curves, we can see that the performance of our proposed system, 4D-8-PSK-TCM-DSTBC, outperforms the corresponding coherent scheme for nearly full practical SNR range. In fact, the 4D-TCM-DSTBC offers 6 dB coding gain for the DSTBC at BER of 10-3 and for lower BER requirement this coding gain is larger.

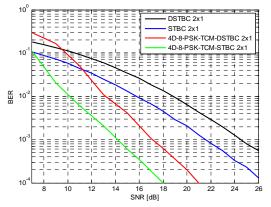


Fig. 3 Performance comparison between concatenating DSTBC with 4D-8-PSK-TCM and uncoded system, Nt = 2; Nr=1

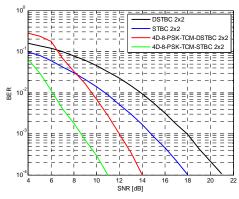


Fig. 4 Performance comparison between concatenating DSTBC with 4D-8-PSK-TCM and uncoded system, Nt = 2; Nr=2

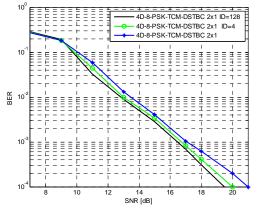


Fig. 5 BER of the proposed 4D-8-PSK-TCM-DSTBC versus SNR for various values of interleaver lengths, Nt = 2; Nr=1.

In order to prevent the appearance of long bursts of errors within the received signal, a symbol interleaver is introduced between the outer coder and the inner coder. As intuitive, the performance of the proposed system can be considerably enhanced when the interleaving depth ID is sufficiently large.

Fig. 5 shows the robustness of the 4D-8-PSK-TCM-DSTBC scheme with various interleaver depths. From this figure, we see clearly that symbol interleaver with ID  $\geq$  4 involves significant robustness for the proposed method. In fact, we remark that when the interleaving depth equals to four frames, the curve at BER=10<sup>-4</sup> is approximately 1 dB better than that without interleaver. When ID=128, the coding gain is not significantly better than the case where ID=4. Thus, we can conclude that the performances of the 4D-8-PSK-TCM-DSTBC scheme are optimal for an interleaver depth equal to 4. According to [10], the optimal interleaver for the 4D-8-PSK-TCM-STBC remains equal to 4.Consequently, regardless to the conventional 3 dB loss of symbol by symbol differential detection, the gain involved by both the multidimensional TCM and also the interleaver with adequate depth the proposed scheme offers attractive coding gain and also competitive BER behavior over severe fading channel.

#### VI. CONCLUSION

In this paper, we propose a concatenation of 4D-8-PSK-TCM scheme in conjunction with differential STBC over slow, flat Rayleigh fading. At first, we compared the performance of the presented scheme to the uncoded DSTBC system. Simulation results showed that this differential method achieves interesting coding gains over quasi static fading channel. A noticeable result is that a very simple differential detection shows 3 dB signal-to-noise penalty, compared to perfect CSI case. However, it is expected that assuming fast fading channels leads to large performance degradation. Our future work will be focused on the decreasing of the 3 dB performance degradation.

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